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On the Range of Permeability in Ferric Conductors

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ON THE RANGE OF PERMEABILITY IN FERRIC CONDUCTORS

BY

CLARENCE HENRY KESSLER

THESIS

FOR

DEGREE OF BACHELOR OF SCIENCE

IN

ELECTRICAL ENGINEERING

COLLEGE OF ENGINEERING

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Clarence Henry Kessler

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IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Electrical Engineering

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ON THE RANGE OF PERMEABILITY IN FERRIC CONDUCTORS

I

INTRODUCTION.

Copper has been practically the only material used in conductors for the transmission of electric energy because of its high electric conductivity. With the introduction of high voltage power transmission the demand for high conductivity of the conductor has become less insistent, and the attention of designers has been called to the mechanical strength of line necessary for long spans. As a result, iron and steel conductors are being considered.

It can be shown experimentally and mathematically that when alternating current is passed through a conductor it does not distribute itself equally over the cross-section, but that the current density is greatest in the outer layers of the wire. Since the inner portion of the conducting wire does not carry its share of current the effective cross-section for current-carrying duty is reduced, and the resistance is increased to a higher value called the effective resistance. In conductors of large cross-section, especially those of magnetic materials, this unequal current distribution causes a resistance loss and voltage drop much larger than those due to direct cur-

rent which utilizes the whole cross-section. In steel conductors the increase of resistance is very appreciable even at low frequency and small sectional area. This phenomenon, known as "skin effect", is the most important problem confronting the engineer in determining the practicability of using steel conductors in alternating current power transmission.

Considerable mathematical investigation of this phenomenon has been made by Maxwell, Lord Rayleigh, Lord Kelvin, and others. All of their solutions involve the value of the permeability of the material with little or nothing said about its numerical value or how to calculate it. Various values for permeability of iron and steel are assumed ranging from 200 to 2,000 with nothing said about how or with what it varies.

It is the purpose of this thesis to determine the value of permeability to be used in calculating the skin effect coefficient of steel wire conductors.

II

MATHEMATICAL THEORY

The general differential equation connecting the current density and radial distance in the cross-section of a conductor is:

$$4\pi\mu\lambda\frac{dI}{dr} = \frac{\partial^2 I}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial I}{\partial r} \qquad (1)$$

where I = current density in the conductor

r = distance of zone from center of conductor

μ = permeability

λ = conductivity

The solution of this equation, given by Maxwell, is:

$$\frac{R_E}{R} = K = 1 + \frac{w^2\mu^2}{12\ r^2} - \frac{w^4\mu^4}{180\ r^4} + \frac{23\ w^6\mu^6}{80640\ r^6} - - \qquad (2)$$

where $R_E = \frac{W}{I^2}$

R = ohmic resistance of the conductor

$w = 2\pi f$

K = skin effect coefficient

r = resistance of conductor in absolute units

W = watts energy consumed

f = frequency

The solution of equation (2) for permeability is very difficult since it is an infinite series of which only a few terms are given here.

For commercial frequencies (20 to 100 cycles) the term $\frac{w^4\mu^4}{180\ r^4}$ is very small compared to the preceding term so it and the following terms may be neglected, leaving the equation in the following simple form:

$$K = 1 + \frac{w^2\mu^2}{12\ r^2} \tag{3}$$

Solving for μ:

$$\mu = \frac{r}{w}\sqrt{12(K-1)} \tag{4}$$

Another solution of this equation which is given by Lord Kelvin depends on two functions called the ber and bei functions. The solution is

$$q = \pi d\sqrt{\frac{2\mu f}{\rho}} \tag{5}$$

The skin coefficient is

$$K = \frac{R_E}{R} = \frac{q}{2} \cdot \frac{\text{ber}q\ \text{bei}'q - \text{bei}q\ \text{ber}'q}{(\text{ber}'q)^2 + (\text{bei}'q)^2} \tag{6}$$

where

$$\text{ber}\ q = 1 + \frac{q^4}{2^2.4^2} + \frac{q^8}{2^2.4^2.6^2.8^2} + \text{----} \tag{7}$$

$$\text{bei}\ q = \frac{q^2}{2^2} - \frac{q^6}{2^2.4^2.6^2} + \text{----} \tag{8}$$

$$\text{ber}'q = \frac{d(\text{ber}q)}{dq} \tag{9}$$

$$\text{bei}'q = \frac{d(\text{bei}\ q)}{dq} \tag{10}$$

d - diameter of conductor in centimeters

f = frequency

ρ = specific resistance in absolute units

μ = permeability

Considering K as a function of q Dr. Magnus McLean has calculated the following table of numerical values for K corresponding to values of q ranging from 0 to 20.

q	$K = \frac{R_E}{R}$	q	$K = \frac{R_E}{R}$
0.0	1.0000	4.5	1.8628
0.5	1.0000	5.0	2.0430
1.0	1.0001	5.5	2.2190
1.5	1.0258	6.0	2.3937
2.0	1.0805	8.0	3.0956
2.5	1.1747	10.0	3.7940
3.0	1.3180	15.0	5.5732
3.5	1.4920	20.0	7.3250
4.0	1.6778		

Plotting these values shows that the critical values of K are from $q = 0$ to $q = 5$. For $q > 5$ the curve is a straight line, showing that K varies directly as q.

Solving equation (4) for μ:

$$\mu = \frac{q^2 \rho}{2\pi^2 d^2 f}$$

From experimental data the value of K is determined. From the table the corresponding value of q is found. Then all the terms of the right hand member are known, and the value of permeability can be obtained.

III

EXPERIMENTAL APPARATUS

The conductors tested in this experiment were:

3/16 inch solid steel, galvanized
3/8 inch solid iron
1/2 inch solid iron

Dimensions and ohmic resistance of each conductor are given on page (10).

Each conductor was bent so as to form a parallel return loop, the distance from inner surface of the conductor to the inner surface of the return conductor being six inches.

Variable room temperature and large line currents caused variable ohmic resistance. The first plan was to observe the conductor temperature by means of several thermometers spaced at equal intervals with the mercury bulbs placed next to the conductor and covered with a thin layer of putty. This method failed in two respects: the temperature changed while the several thermometers were being read, and the putty covering, by its absorption and retention of heat, caused a lag in the indications of the thermometers. The fall of potential method of measuring resistance was finally adopted. With a direct current ammeter and low reading voltmeter the data for calculation of ohmic resistance was obtained. In case the resistance had changed, time was given for the conductor to cool or else it was heated with a large alternating current until the average resistance was obtained again.

The power consumed by the conductor was measured by General Electric wattmeter dynamometer No. 181928. The sensibility of the instrument was a deflection of 500 millimeters on a scale one meter from the mirror for a power measurement of .128 watts. For measurements of power greater than this resistances were put in series with the potential coil of the instrument. These resistances ranged in value from 0 to 60,000 ohms. Since the instrument was not direct reading it was necessary to calibrate it for each value of resistance used. All the calibrations were made with direct current power and checked from time to time during the test so as to detect any possible disturbance of the instrument. The zero reading of the dynamometer was obtained frequently as a precaution against accidental movement of the instrument with a consequent change in calibration due to a different relative position of the suspended and fixed coils.

The current was measured with Weston ammeters designed for frequencies under 133 cycles per second. Each ammeter was calibrated with a standard instrument at frequent intervals during the tests. The frequency was measured with a Biddle frequency meter of the vibrating reed type.

The power at 25 and 40 cycles was furnished by a Thomson Houston smooth core generator, No. 683, Class A35, 1100 volts, 1500 R.P.M. The voltage was stepped down to about 60 volts through a General Electric transformer, No. 745828, Type H60, Form K, 1100/2200 110/220 volts, 5 K.W.,

with the high tension coils in parallel and the low tension coils in series. Rheostats in series with the conductor gave line current control, and rheostatic control of the armature and field current of the driving motor assured constant frequency. Since the motor could not attain the speed necessary for 90 cycles, both the 60 and 90 cycle runs were made on General Electric generator No. 95584, Type AHB, Class 6-7. 5, Form A, 20 amperes, 220 volts, 1200 R.P.M. The six armature coils are spaced 30 electrical degrees apart on the armature. No transformer was used in this set up, rheostats in the generator field circuit giving the necessary voltage regulation. The smooth core generator gave a sine voltage wave; the wave from the General Electric generator was also very free from harmonics because of the series connection of the armature coils.

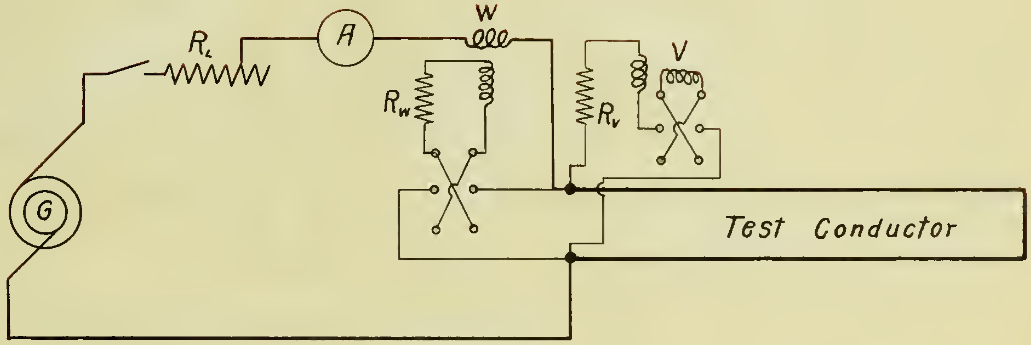


Diagram of Connections

IV

EXPERIMENTAL RESULTS

The results of the experimental tests are given in the form of curves from page (11) to page (28). The observed data is tabulated on the log sheets within the same pages. From the observed values of watts consumed and line current the effective resistance $R_E = (\frac{W}{I^2})$ was calculated for each value of line current and the values plotted. From a smooth curve drawn through the points new values of effective resistance R'_E were obtained, and by the aid of the K-q curves on page (29), permeability values were calculated. The permeability values were plotted as ordinates and amperes per square centimeter I_0 as abcissa, giving the permeability-current density curves at constant frequency, shown on pages (16), (22), and (28) for the different conductors tested.

Constants of Conductors.

Conductor	Length	Diameter in inches	Resistance in ohms.	ρ c.g.s. units.	$\frac{\mu f}{\rho^2}$
3/16" Steel	64'10"	.190	.1366	12,650	2750
3/8" Iron	63'9"	.380	.041	16,040	810
1/2" Iron	54'9"	.500	.022	16,900	532

3/16 INCH SOLID STEEL CONDUCTOR. 25 CYCLES.

I_D	I	W	R_E	R'_E	K	q	μ
5.47	1			.1430	1.045	1.70	319
10.93	2			.1505	1.100	2.12	495
16.40	3	1.46	.1622	.1595	1.170	2.48	675
21.85	4	2.74	.1712	.1715	1.255	2.81	870
27.35	5	4.66	.1864	.1875	1.375	3.15	1095
32.80	6	7.40	.2055	.2040	1.495	3.48	1335
38.25	7	10.24	.2126	.2130	1.560	3.66	1475
43.75	8	13.80	.2156	.2175	1.595	3.76	1560
49.20	9	17.60	.2170	.2205	1.615	3.82	1610
54.70	10	22.30	.2230	.2225	1.640	3.86	1640
60.10	11			.2225	1.640	3.86	1640
65.60	12	32.10	.2230	.2220	1.625	3.84	1625
71.00	13			.2190	1.605	3.78	1575
76.50	14	42.40	.2160	.2160	1.580	3.71	1515
82.00	15	47.40	.2110	.2113	1.550	3.63	1450

3/16 INCH SOLID STEEL CONDUCTOR.40 CYCLES.

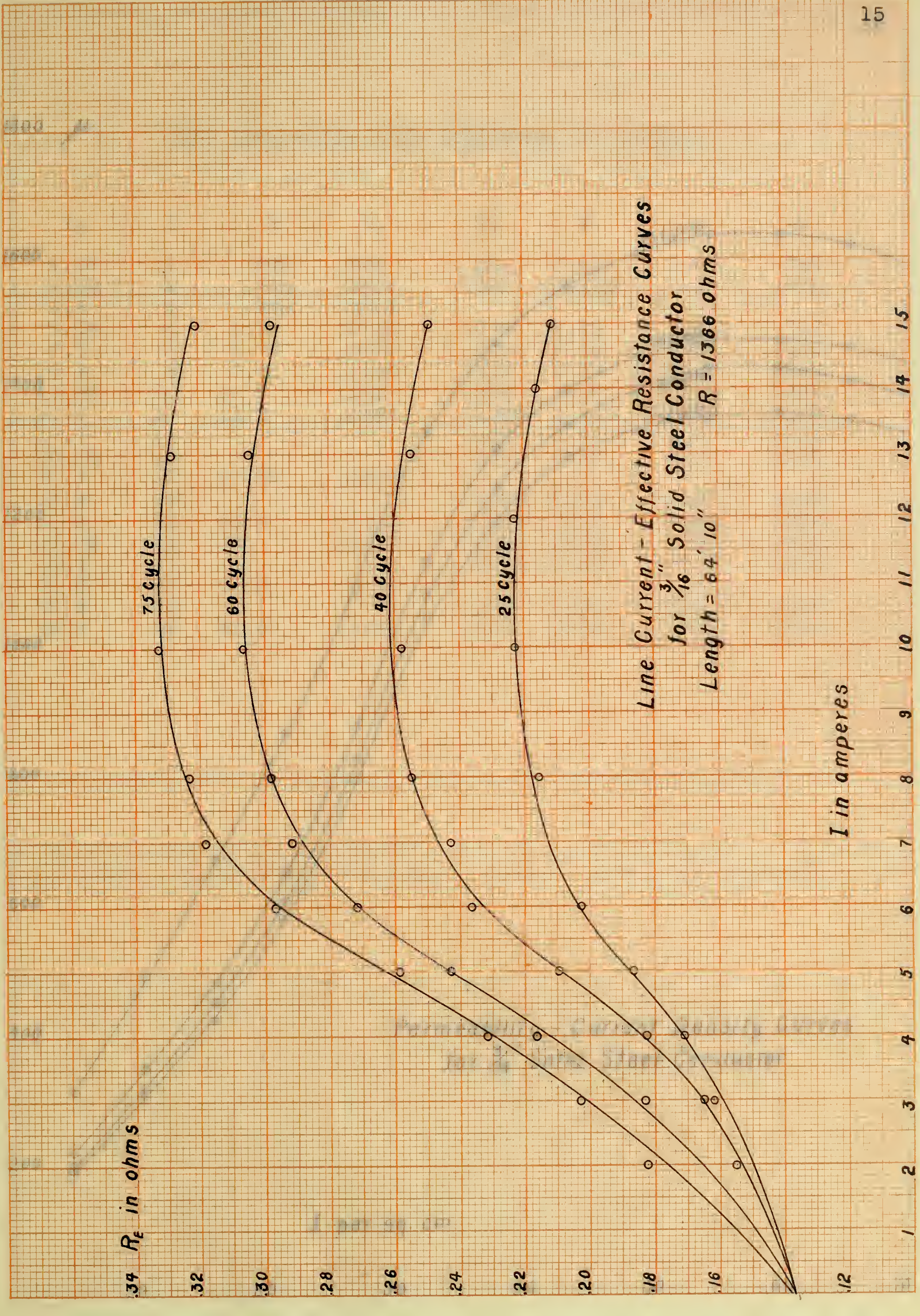
I_0	I	W	R_E	R'_E	K	q	μ
5.47	1			.1441	1.055	1.80	223
10.93	2	.62	.1550	.1538	1.125	2.26	351
16.40	3	1.48	.1645	.1655	1.210	2.65	485
21.85	4	2.92	.1825	.1840	1.350	3.08	655
27.35	5	5.24	.2096	.2090	1.530	3.58	880
32.80	6	8.52	.2366	.2330	1.705	4.06	1135
38.25	7	11.90	.2430	.2470	1.810	4.36	1310
43.75	8	16.30	.2548	.2550	1.865	4.51	1400
49.20	9			.2590	1.895	4.59	1450
54.70	10	25.80	.2580	.2609	1.910	4.63	1475
60.10	11			.2610	1.910	4.63	1475
65.60	12			.2596	1.900	4.61	1465
71.00	13	43.00	.2543	.2573	1.885	4.57	1435
76.50	14			.2540	1.860	4.50	1395
82.00	15	56.30	.2494	.2494	1.825	4.40	1330

3/16 INCH SOLID STEEL CONDUCTOR. 60 CYCLES.

I_a	I	W	R_E	R'_E	K	q	μ
5.47	1	.38	.1688	.1498	1.096	2.10	202
10.93	2	.73	.1824	.1658	1.214	2.66	325
16.40	3	1.65	.1834	.1864	1.365	3.14	455
21.85	4	3.48	.2174	.2120	1.552	3.65	610
27.35	5	6.50	.2600	.2430	1.780	4.26	830
32.80	6	9.80	.2720	.2715	1.990	4.85	1080
38.25	7	14.30	.2920	.2885	2.110	5.22	1250
43.75	8	19.10	.2985	.2983	2.185	5.42	1345
49.20	9			.3035	2.220	5.53	1400
54.70	10	30.70	.3070	.3060	2.240	5.60	1435
60.10	11			.3063	2.245	5.60	1435
65.60	12			.3055	2.235	5.55	1410
71.00	13	51.50	.3046	.3033	2.220	5.50	1385
76.50	14			.3000	2.195	5.42	1345
82.00	15	67.00	.2980	.2955	2.165	5.35	1310

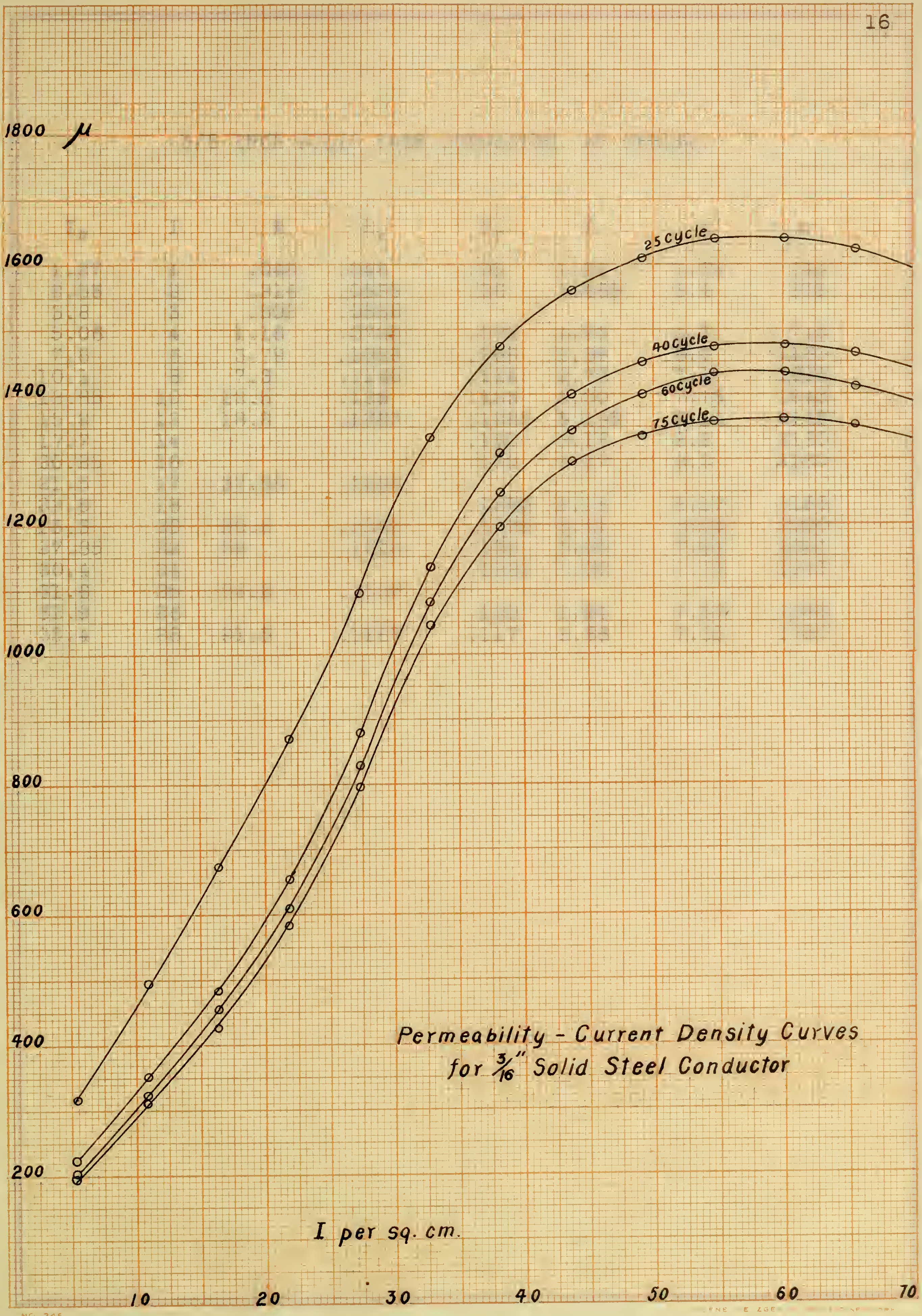
3/16 INCH SOLID STEEL CONDUCTOR.75 CYCLES.

I_0	I	W	R_E	R'_E	K	q	μ
5.47	1			.1552	1.133	2.304	195
10.93	2	.73	.1825	.1762	1.286	2.91	311
16.40	3	1.83	.2033	.2008	1.465	3.415	429
21.85	4	3.71	.2320	.2295	1.675	3.985	584
27.35	5	6.48	.2590	.2628	1.918	4.655	796
32.80	6	10.70	.2974	.2960	2.160	5.33	1043
38.25	7	15.60	.3185	.3143	2.294	5.71	1197
43.75	8	2o.7o	.3235	.3250	2.372	5.94	1297
49.20	9			..3298	2.407	6.03	1336
54.70	10	33.30	.3330	.3318	2.421	6.08	1359
60.10	11			.3323	2.426	6.09	1362
65.60	12			.3315	2.420	6.075	1357
71.00	13	55.50	.3284	.3290	2.400	6.01	1327
76.50	14			.3260	2.380	5.96	1306
82.00	15	72.30	.3210	.3230	2.350	5.87	1266



Line Current - Effective Resistance Curves for 3/16" Solid Steel Conductor

Length = 64' 10" R = 1366 ohms



Permeability - Current Density Curves for $\frac{3}{16}$" Solid Steel Conductor

3/8 INCH SOLID IRON CONDUCTOR. 25 CYCLES.

I_D	I	W	R_E	R'_E	K	q	μ
1.27	1	.048	.048	.05	1.22	2.69	245
2.53	2	.214	.0535	.06	1.465	3.4	375
3.8	3	.502	.0558				
5.06	4	1.16	.0725	.079	1.93	4.7	715
7.6	6	3.79	.1052	.098	2.39	6.0	1165
10.1	8	7.3	.1140	.114	2.78	7.1	1635
12.65	10	12.5	.125	.123	3.00	7.75	1945
15.2	12	18.2	.1263	.1285	3.135	8.14	2145
17.7	14			.130	3.17	8.2	2180
20.25	16			.130	3.17	8.2	2180
21.5	17	37.55	.1298				
22.8	18			.1290	3.15	8.17	2155
25.3	20	50.5	.1262	.1275	3.11	8.04	2090
27.85	22	59	.1218	.125	3.05	7.91	2025
30.4	24			.1225	3.00	7.75	1945
31.6	25	74.2	.1187				
32.9	26			.120	2.94	7.57	1850
35.4	28	91.5	.1167	.117	2.86	7.35	1750

3/8 INCH SOLID IRON CONDUCTOR. 40 CYCLES.

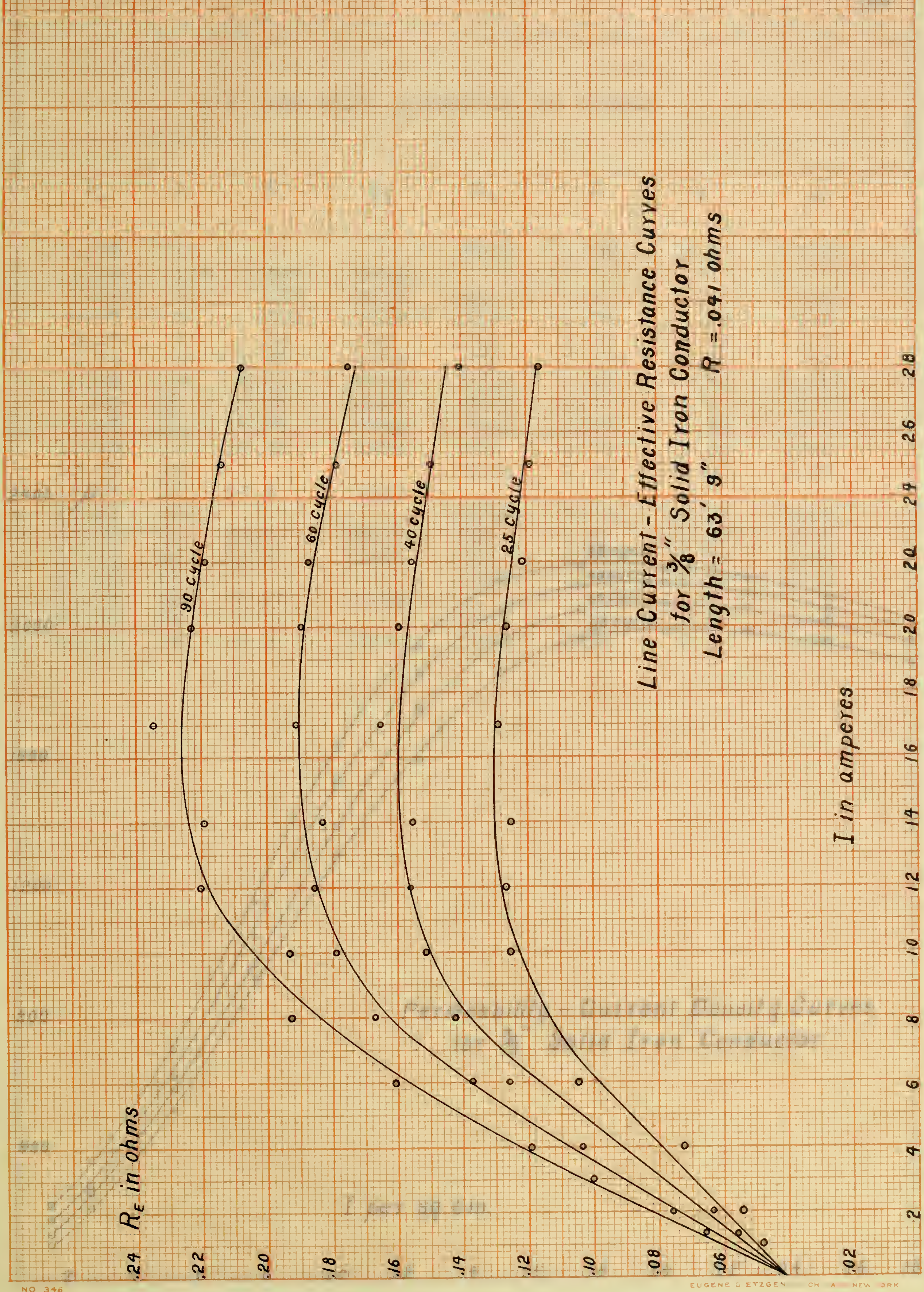
I_D	I	W	R_E	R_E'	K	q	μ
1.27	1			.054	1.318	2.98	190
2.53	2	.253	.0633	.0665	1.62	3.83	297
3.8	3	.61	.0678				
5.06	4	1.23	.0769	.092	2.245	5.59	632
7.6	6	4.55	.1263	.116	2.815	7.25	1060
10.1	8	9.1	.1422	.138	3.37	8.7	1530
12.65	10	15.1	.151	.150	3.65	9.6	1860
15.2	12	22.45	.1558	.156	3.81	10.05	2050
17.7	14	30.5	.1556	.159	3.88	10.25	2130
20.25	16			.1595	3.89	10.28	2140
21.5	17	47.7	.1650				
22.8	18			.158	3.85	10.20	2110
25.3	20	63.8	.1595	.1565	3.82	10.08	2060
27.85	22	75	.1549	.154	3.75	9.91	1990
30.4	24			.151	3.68	9.71	1910
31.6	25	92.95	.1487				
32.9	26			.148	3.61	9.5	1830
35.4	28	109.6	.1398	.144	3.515	9.22	1720

3/8 INCH SOLID IRON CONDUCTOR. 60 CYCLES.

I_0	I	W	R_E	R'_E	K	q	μ
1.27	1			.058	1.415	3.26	144
1.65	1.3	.112	.0663				
2.53	2	.296	.0759	.0755	1.84	4.44	276
5.80	3	.923	.1025				
5.06	4	1.66	.1037	.1065	2.6	6.61	590
7.6	6	4.95	.1375	.1380	3.25	8.47	970
10.1	8	10.7	.1672	.163	3.975	10.54	1390
12.65	10	17.9	.179	.176	4.28	11.42	1760
15.2	12	26.7	.1853	.1845	4.5	12.04	1960
17.7	14	35.8	.1827	.189	4.61	12.35	2060
20.25	16			.190	4.63	12.4	2080
21.5	17	58	.2006				
22.8	18			.1895	4.62	12.35	2060
25.3	20	75.9	.1897	.189	4.59	12.3	2020
27.85	22	90.4	.1867	.185	4.51	12.03	1955
30.4	24			.181	4.42	11.79	1870
31.6	25	111.5	.1785				
32.9	26			.177	4.32	11.51	1790
35.4	28	136.9	.1746	.1725	4.21	11.21	1700

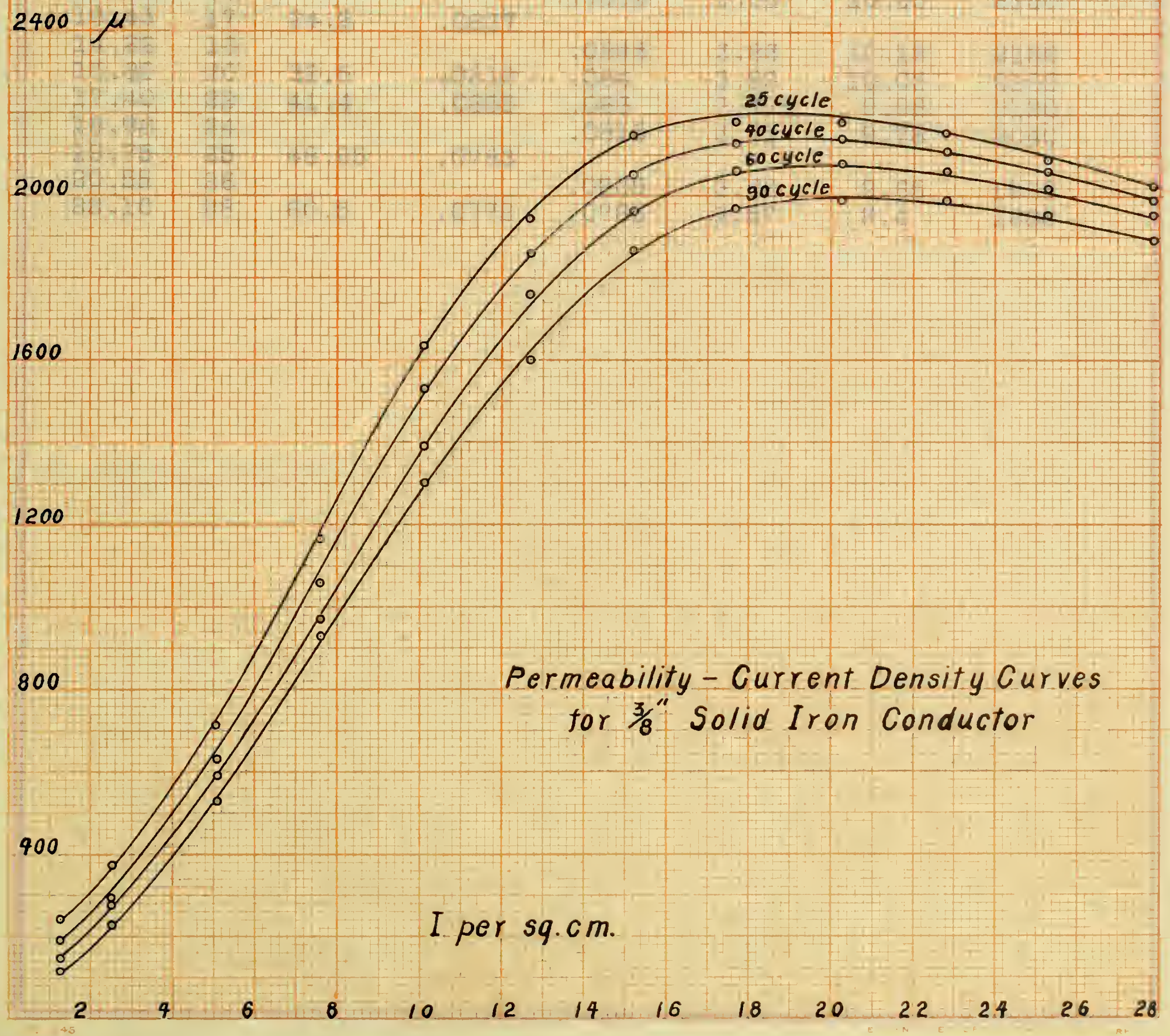
3/8 INCH SOLID IRON CONDUCTOR. 90 CYCLES.

I_0	I	W	R_E	R_E'	K	q	μ
1.27	1	.066	.066	.0625	1.525	3.58	115
2.53	2	.235	.0588	.0835	2.035	5.00	225
3.8	3	.900	.100				
5.06	4	1.91	.1193	.122	2.975	7.69	530
7.6	6	5.81	.1613	.1575	3.84	10.15	930
10.1	8	12.3	.1923	.184	4.49	12.0	1300
12.65	10	19.3	.193	.203	4.95	13.33	1600
15.2	12	31.85	.221	.218	5.32	14.4	1865
17.7	14	43.05	.2196	.224	5.47	14.8	1970
20.25	16			.225	5.49	14.86	1990
21.5	17	67.6	.234				
22.8	18			.225	5.49	14.86	1990
25.3	20	89.5	.2237	.223	5.44	14.73	1955
27.85	22	105.6	.218	.22	5.36	14.5	1895
30.4	24			.216	5.26	14.2	1815
31.6	25	133	.213				
32.9	26			.212	5.17	13.96	1755
35.4	28	162.6	.2075	.207	5.05	13.72	1700



Line Current - Effective Resistance Curves
for $3/8$" Solid Iron Conductor
Length = 63' 9" R = .041 ohms

NO 346
EUGENE DIETZGEN CO. CHICAGO NEW YORK



Permeability - Current Density Curves for $\frac{3}{8}$" Solid Iron Conductor

1/2 INCH SOLID CONDUCTOR. 25 CYCLES.

I_D	I	W	R_E	R'_E	K	q	μ
.79	1			.0275	1.25	2.8	166
1.18	1.5	.067	.0298				
1.58	2	.126	.0315	.033	1.5	3.5	260
2.37	3	.293	.0326	.038	1.73	4.15	365
3.16	4	.64	.040	.044	2.00	4.88	515
4.74	6	1.82	.0506	.054	2.455	6.20	840
6.32	8	4.01	.0626	.0645	2.93	7.55	1210
7.90	10	6.81	.0681	.0725	3.3	8.6	1570
9.48	12	10.75	.0746	.0785	3.57	9.38	1870
11.05	14	16.45	.0839	.082	3.73	9.85	2060
12.65	16			..0835	3.80	10.05	2150
13.43	17	24.2	.0837				
14.22	18			.0845	3.83	10.12	2180
15.80	20	32.5	.0812	.084	3.82	10.08	2160
17.40	22	41.4	.0855	.083	3.775	9.98	2120
18.96	24			.0815	3.71	9.78	2040
19.75	25	48.95	.0783				
20.55	26			.0805	3.66	9.65	1980
22.10	28	60.5	.0772	.0785	3.57	9.4	1880

1/2 INCH SOLID IRON CONDUCTOR. 40 CYCLES.

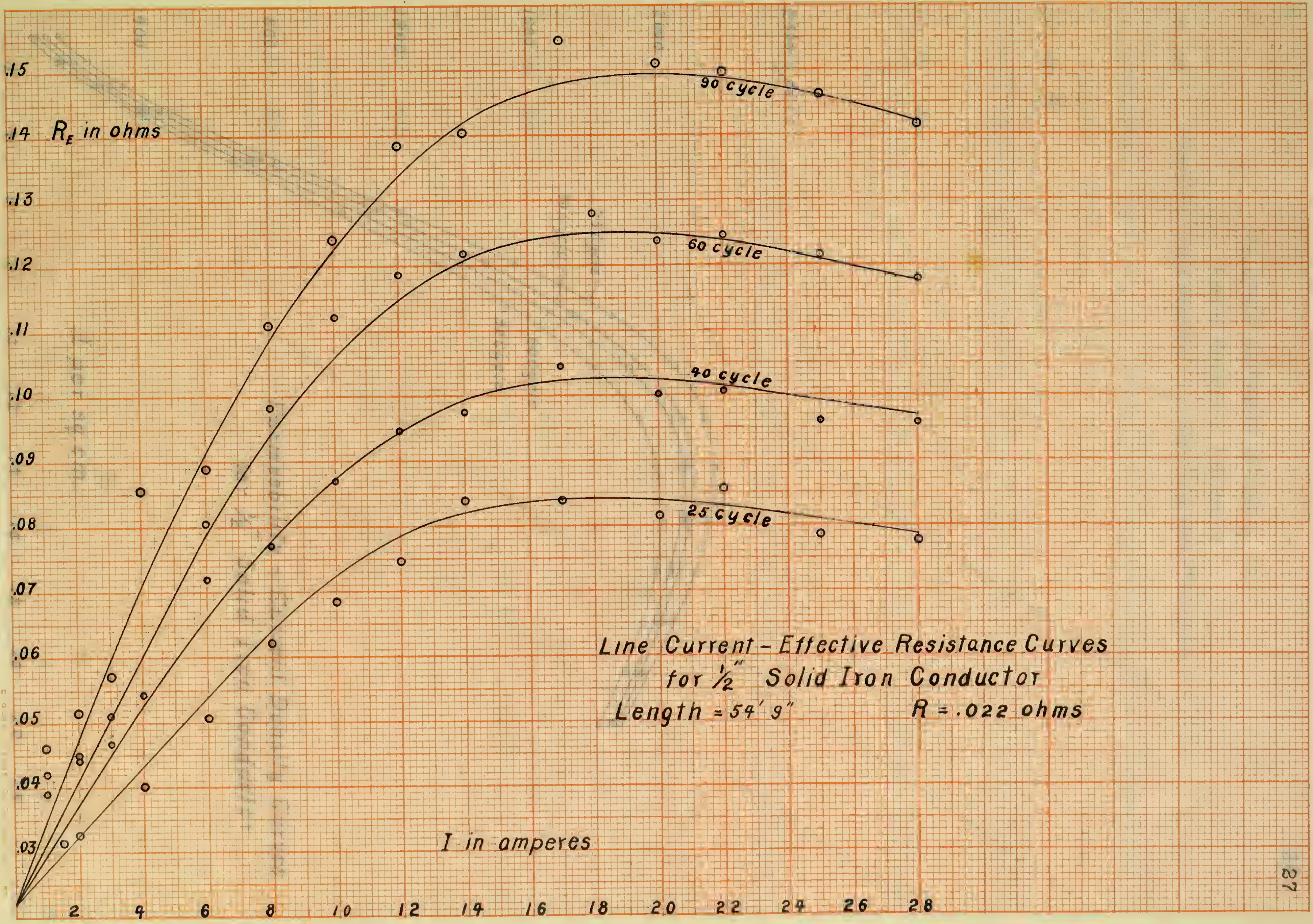
$I_\square$	I	W	R_E	R'_E	K	q	μ
,79	1	.039	.039	.030	1.365	3.12	130
1.58	2	.179	.0448	.0375	1.705	4.06	219
2.37	3	.434	.0482	.0455	2.07	5.1	346
3.16	4			.0525	2.385	6.0	480
4.74	6	2.59	.072	.0655	2.98	7.7	790
6.32	8	4.93	.077	.0785	3.57	9.38	1160
7.90	10	8.7	.087	.0875	3.98	10.55	1480
9.48	12			.0945	4.3	11.46	1750
11.05	14			.0995	4.52	12.15	1950
12.65	16			.102	4.64	12.25	2085
13.43	17	30.2	.1045				
14.22	18			.1025	4.65	12.55	2100
15.80	20	40.1	.1002	.1027	4.67	12.5	2085
17.40	22	48.7	.1006	.1015	4.61	12.35	2040
18.96	24			.0998	4.54	12.3	2010
19.75	25	61	.0960				
20.55	26			.0982	4.51	12.1	1950
22.10	28	75	.0957	.097	4.42	11.8	1860

1/2 INCH SOLID IRON CONDUCTOR. 60 CYCLES.

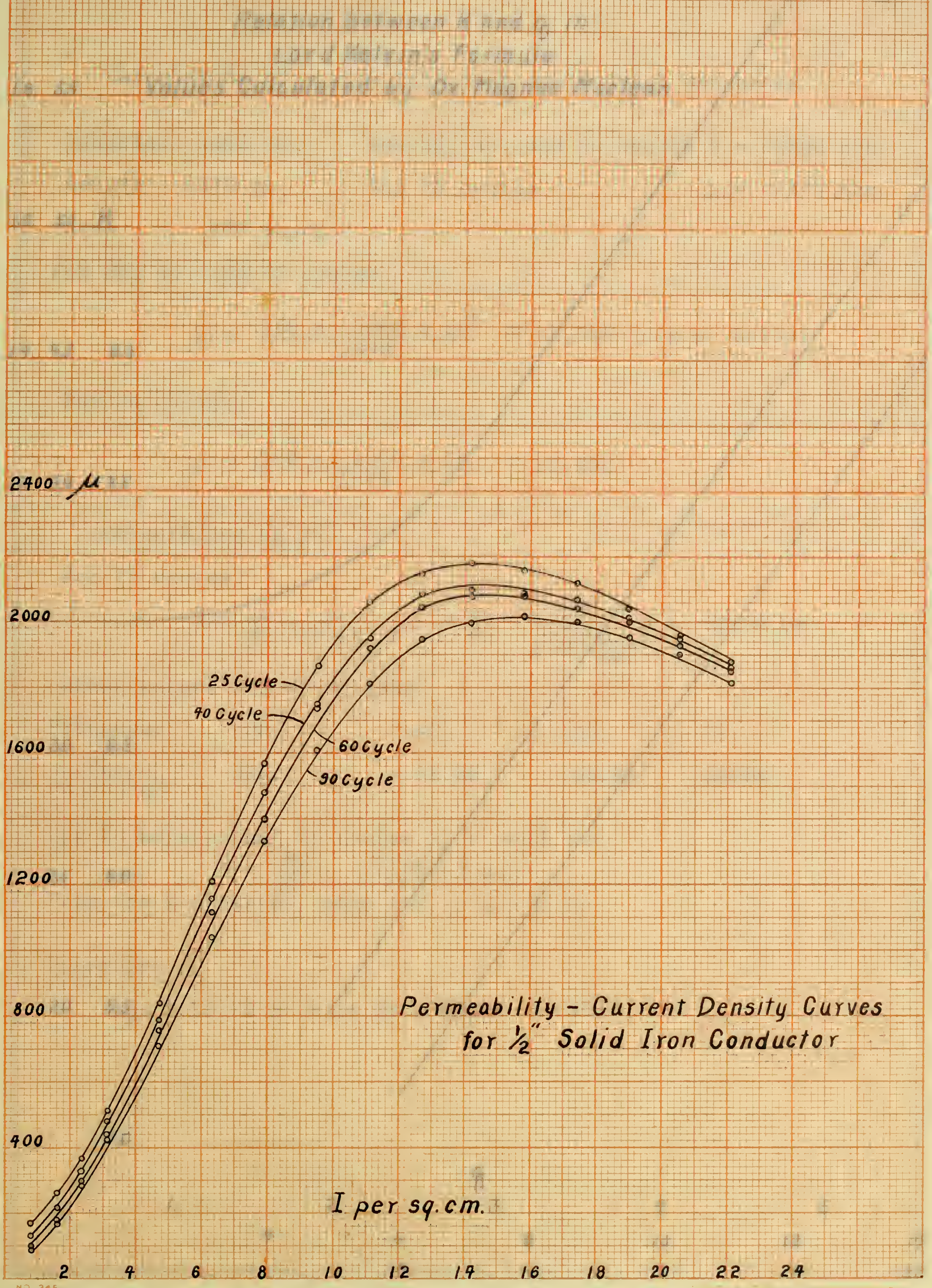
I_p	I	W	R_E	R'_E	K	q	μ
.79	1	.042	.042	.032	1.455	3.38	100
1.58	2	.180	.045	.0415	1.885	4.48	178
2.37	3	.421	.0468	.051	2.32	5.8	298
3.16	4	.78	.0488	.0605	2.75	7.04	440
4.74	6			.079	3.59	9.45	760
6.32	8	6.28	.0981	.0945	4.24	11.45	1120
7.90	10	11.2	.112	.1065	4.84	13.00	1400
9.48	12	17.1	.1187	.115	5.21	14.05	1750
11.05	14	23.9	..1218	.1205	5.46	14.75	1920
12.65	16			.124	5.64	15.2	2045
13.43	17	37.0	.1280				
14.22	18			.125	5.68	15.35	2080
15.80	20	49.5	.1237	.125	5.68	15.35	2070
17.40	22	60.3	.1245	.1245	5.65	15.3	2070
18.96	24			.122	5.55	15.04	2000
19.75	25	76	.1216				
20.55	26			.120	5.45	14.75	1930
22.10	28	92.4	.1178	.1175	5.34	14.45	1850

1/2 INCH SOLID IRON CONDUCTOR. 90 CYCLES.

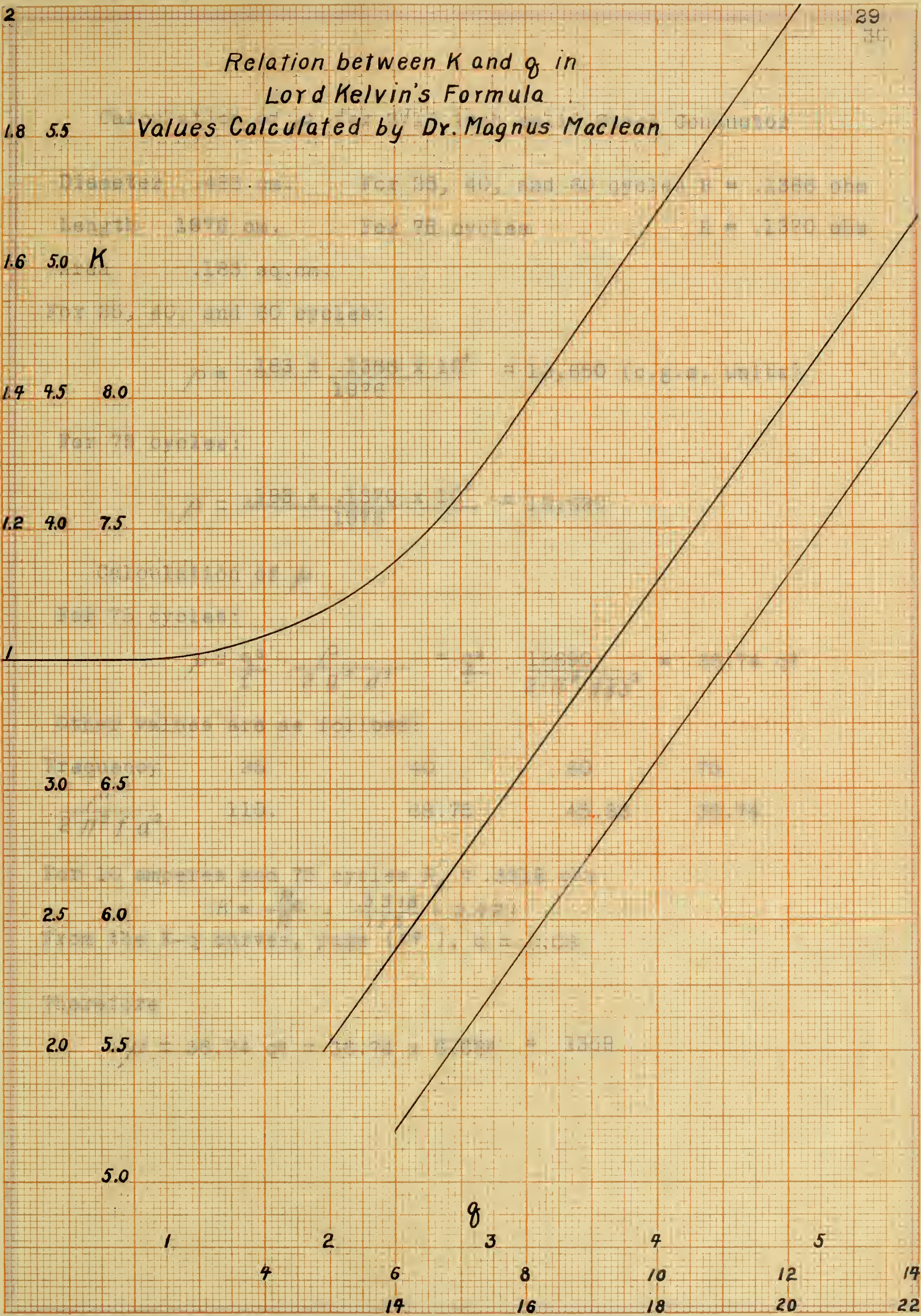
I_p	I	W	R_E	R'_E	K	q	μ
.79	1	.046	.046	.0355	1.613	3.84	87
1.58	2	.206	.0515	.0475	2.16	5.32	168
2.37	3	.513	.0510	.0595	2.705	6.9	282
3.16	4	1.045	.0853	.0715	3.25	8.45	425
4.78	6	3.2	.0889	.091	4.14	10.95	710
6.32	8	7.1	.111	.109	4.95	13.25	1040
7.90	10	12.4	.124	.1225	5.56	15.00	1335
9.48	12	19.9	.1382	.1335	6.29	17.05	1610
11.05	14	27.5	.1402	.142	6.45	17.5	1815
12.65	16			.1465	6.65	18.15	1950
13.43	17	44.7	.1546				
14.22	18			.1488	6.75	18.4	2000
15.80	20	60.4	.151	.1490	6.77	18.5	2020
17.40	22	72.5	.1497	.1488	6.75	18.4	2000
18.96	24			.1475	6.7	18.18	1955
19.75	25	91.3	.461				
20.55	26			.145	6.59	17.9	1900
22.10	28	111	.1416	.142	6.45	17.5	1815



Line Current - Effective Resistance Curves
for 1/2" Solid Iron Conductor
Length = 54' 9" R = .022 ohms



Permeability - Current Density Curves for ½" Solid Iron Conductor



Relation between K and q in Lord Kelvin's Formula

Values Calculated by Dr. Magnus Maclean

Calculation of ρ for 3/16 inch solid Steel Conductor

Diameter .483 cm. For 25, 40, and 60 cycles R = .1366 ohm

Length 1976 cm. For 75 cycles R = .1370 ohm

Area .183 sq.cm.

For 25, 40, and 60 cycles:

$$\rho = \frac{.183 \times .1366 \times 10^9}{1976} = 12,650 \text{ (c.g.s. units)}$$

For 75 cycles:

$$\rho = \frac{.183 \times .1370 \times 10^9}{1976} = 12,690$$

Calculation of μ

For 75 cycles:

$$\mu = \frac{q^3}{f} \cdot \frac{\rho}{2\,\pi^2\,d^2} = \frac{q^3}{f} \cdot \frac{12690}{2 \cdot \pi^2 \cdot \overline{.483}^2} = 36.74\,q^3$$

Other values are as follows:

Frequency	25	40	60	75
$\frac{\rho}{2\,\pi^2 f\,d^2}$	110.	68.75	45.83	36.74

For 10 amperes and 75 cycles $R_E' = .3318$ ohm.

$$K = \frac{R_E}{R} = \frac{.3318}{.173} = 2.421$$

From the K-q curves, page (29), q = 6.08

Therefore

$$\mu = 36.74\,q^3 = 36.74 \times \overline{6.08}^3 = 1359$$

Key to Notation.

R = ohmic resistance

R_E = effective resistance as calculated from $\frac{W}{I^2}$

R'_E = " " taken from curves of R_E

$K = \frac{R_E}{R}$ = skin effect coefficient

W = watts consumed by the conductor

I = line current flowing in the conductor

I_a = amperes per square centimeter, calculated from total area

f = frequency in cycles per second

q = variable in Lord Kelvin's formula

μ = permeability

ρ = conductivity in absolute units

V

CONCLUSIONS

The permeability-current density curves, (pages 16 to 28) indicate that in the conductors tested the permability increased rapidly as a linear function of current density until nearly the maximum permeability was reached. After the maximum was passed the permeability decreased linearly but much more slowly. The following equations are approximate expressions which give values of permeability within 10 per cent of those indicated on the curves.

For 3/16 inch solid steel conductors:

$\mu = 38\, I_o + 100$ $\quad I_o = 3 - 35$ $\quad$ 25 cycles
$\mu = 32.5\, I_o$ $\quad I_o = 3 - 35$ $\quad$ 60 cycles

For 3/8 inch solid iron conductor:

$\mu = 150\, I_o - 50$ $\quad I_o = 4 - 13$ $\quad$ 25 - 60 cycles
$\mu = 1100 + 63\, I_o$ $\quad I_o = 13 - 17$
$\mu = 2400 - 15\, I_o$ $\quad I_o = 17 - 28$

For 1/2 inch solid iron conductor:

$\mu = 193\, I_o - 100$ $\quad I_o = 3 - 11$ $\quad$ 25 - 60 cycles
$\mu = 35\, I_o + 1600$ $\quad I_o = 11 - 15$
$\mu = 2700 - 37\, I_o$ $\quad I_o = 15 - 22$

The maximum permeability obtained in the steel was about 1650, the maximum in the iron about 2200. The tests indicate an increase of permeability with a decrease of frequency, and an increase of permeability with an increase of specific resistance.

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